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Exploring the Optimal Curves of Intersection for Enhanced 3D Shape Designing: A Comprehensive Study on Geometric Optimization Techniques

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Abstract

Surface and solid intersection is emerging swiftly in many applications of computer graphics, engineering and mathematical models. In engineering science, several algorithms are produced for some unique purposes like intersection of prism/cone, cone/cone, prism/cylinder etc. All such procedures used different 3D drawing tools for creating geometrical interpretation of their required models. Surface intersection deals with numerous algorithms in computational and applied mathematics, needed for intersection of different surfaces such as rational polynomial parametric, implicit algebraic etc. In this paper, a novel surface intersection approach is proposed which works for all types of intersection like; parametric-parametric, explicit-explicit, implicit-explicit surfaces. This scheme includes a method for determination of significant points (boundary and turning points) from surface intersection sequence points. Optimal curve is fitted to these points by rational quadratic spline and a soft computing technique Genetic Algorithm (GA) is applied for attaining optimized curve, by allotting specific shape parameter values in the depiction of rational quadratic spline. However, some experimental results show the efficiency and robustness of this technique.

Keywords: Significant points, Approximation, Curve fitting, Genetic Algorithm, Rational quadratic spline.

1. Introduction

Engineers are dealing "solid and surface" intersection problems with the help of CAD (Computer Aided Drafting) procedure in many fields of engineering. Layouts of different machine components in mechanical engineering, plans of buildings in civil engineering and distribution of power plants in electrical engineering are prepared through this procedure. However, Surface intersection plays an important role in CAD/CAM, Computer Visualization, Animation and Solid Modeling. It is also applicable in medical field; a lot of changes can be done in human beings either it is related to face surgeries or other surgeries for beautifying themselves.

In this era of computer science, technology has growing faster; 3D objects can be modeled into "triangulated meshes" or "spline patches" called as explicit surfaces [12], which are easily to manipulate by graphic designers. These types of representation are not well suited for fitting surfaces to 2d or 3d data points and computerized modeling. For this purpose, implicit surfaces are used. Most of the computational problems related to visualization and computer graphics are more likely close to surface manipulation. Several efforts have done for parametric surface manipulation which needs to find values of parameters of concerned points on surface [8]. Evaluating intersection between two surfaces has been an interesting field for researchers from last few decades. Many methods have been discussed in literature for surface intersection [7, 9, 13] but all of them have their shortcomings and advantages, because iterations are used in almost all methods and even small error in each iteration would give a large amount of error at the end, which makes the efficiency of procedure slower.

The SSI (Surface-Surface Intersection) problems in CAGD are used for solving surface modeling tasks, which can be occurred from intersection of two parametric surfaces. The tracing of parametric representation of surface is easier, it describes the closed images but it would be difficult if any point lies on surface, in that case implicit surface is used. Also in [1] two robust and efficient techniques are used for such type of cases. The main difference between these techniques is, first is applied when one surface is parametric and other is implicit, and second is convenient when successive projection of any one surface to another is given. Most of the researchers give some convergence algorithms which are used to find the least minimum distance and error [9, 14] between two parametric or two implicit surfaces. These are also independent of preliminary values. Though, the essentials of approximation of any intersection curve is, it should be topologically reliable and numerically accurate curve [2] is presented which is more closely to exact curve.

Numerous soft computing techniques are given in literature like fuzzy logic (FL), neural network (NN), genetic algorithm (GA), simulated annealing (SA) etc [3], but mostly used technique is genetic algorithm due to its specific potential of solving

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complex, highly non-linear and multi-dimensional problems of engineering. Also genetic Algorithm (GA) is used in this research work which is a powerful tool to find optimized solution. Curve fitting is done by rational quadratic.

The organization of this paper is given as: Section 2 describes the methodology of proposed scheme. Section 3 is about rational quadratic function and the proposed algorithm along with examples demonstrated in Section 4. The paper is concluded in Section 5.

2. Methodology

- First step in proposed work is to find out the intersection between surfaces. Lots of pair of surfaces is given in literature for intersection [7] like IA/IA (IA means Implicit Algebraic), RPP/RPP (RPP means Rational Polynomial Parametric), RPP/IA etc but in this paper explicit-explicit, implicit-explicit, parametric-parametric surface intersections are used, extensively preferred by researchers nowadays.
- Second step is to calculate all characteristic points through proper organized method.
- In last step rational quadratic with GA is applied for good approximated results by finding optimal values of parameters described in rational quadratic function.

3. Rational Quadratic Spline function

In this section a general rational quadratic spline function is to be considered [5], with control points V_i^*, Z_i , W_i^* and r_i is shape parameter of i^{th} piece. The conic passes through V_i^* and W_i^* while conic shape is affected by Z_i .

$$P_i\left(t\right) = \frac{V_i^*(1-\theta)^2 + r_i Z_i\left(1-\theta\right)\theta + W_i^*\theta^2}{(1-\theta)^2 + r_i\left(1-\theta\right)\theta + \theta^2}$$

The above interpolant can be decomposed into two segments [4], first conic interpolates F_i and Z_i.Also it lies in convex hull of F_i , V_i^*, Z_i .

$$P_i\left(t\right) = \frac{F_i\left(1-\theta\right)^2 + r_i V_i^*\left(1-\theta\right)\theta + Z_i\theta^2}{(1-\theta)^2 + r_i\left(1-\theta\right)\theta + \theta^2} \tag{1}$$

Similarly second conic lies in convex hull of Z_i , W_i^* , F_{i+1} and passes through Z_i and F_{i+1}.

$$P_i^*\left(t\right) = \frac{Z_i\left(1-\theta^*\right)^2 + r_i W_i^*\left(1-\theta^*\right)\theta^* + F_{i+1}{\theta^*}^2}{(1-\theta^*)^2 + r_i\left(1-\theta^*\right)\theta^* + {\theta^*}^2} \tag{2}$$

Conic (1) and (2) should satisfy the following values of unknown control points.

$$V_i^* = F_i + \frac{h_i D_i}{2r_i}$$

$$W_i^* = F_{i+1} - \frac{h_i D_{i+1}}{2r_i}$$

$$Z_i = \frac{V_i^* + W_i^*}{2} = \frac{F_i + F_{i+1}}{2} + \frac{h_i}{4r_i}\left(D_i - D_{i+1}\right)$$

3.1 Genetic Algorithm (GA)

Genetic Algorithms (GAs) are evolutionary-based techniques. They are usually used to develop population of chromosomes through different iterations [3,5] but in this paper it is used for optimization which doesn't need any derivative of functions given in problem. GA is very authentic tool to find optimal results with rational quadratic Spline. It overcome the weakness of speed and feed rate in optimizing as compared to other soft computing techniques. GA helps to choose best optimal value for shape parameter r_i (given in description of rational quadratic spline) from different choices.

GA gives quick answers to the problem due to its robust proficiency and accuracy. The main goal of GA is to pick up those values of parameter r_i from collection of values which minimizes the error. The process of iteration would be repeated until the best approximated results would be achieved.

3.2 Proposed Scheme and Optimal Rational Quadratic Functions

In this section the whole process of finding optimal rational quadratic functions with the help of Genetic Algorithm is discussed.

Suppose $P_{i,j} = (x_{ij}, y_{ij})$ for $i = 1, 2, \ldots, n$, $j = 1, 2, ..., m_i$, be given set of data segments. Then the squared sums $(S_i's)$ of distance among $P_{i,j}'s$ and $P\left(t_j\right)$'s (parametric points) on the curve can be determined as $S_i = \sum_{j=1}^{m_i}\left[P_i\left(u_{i,j}\right) - P_{i,j}\right]^2$ $i = 1, 2, \ldots, n$, where chord length parameterization is used to parameterize $u's$.

Figure 1: Explicit surface-1

Conic 1

When rational quadratic (1) is used for conic then error can be defined as:

$$S_i = \sum_{j=1}^{m_i} \left[P_i\left(u_{i,j}\right) - P_{i,j}\right]^2 i = 1, 2, \ldots, n$$

with same specifications as given above.

Conic 2

Similarly, when conic is declared by rational quadratic (2), then squared sum can be defined as:

$$S_i^* = \sum_{j=1}^{m_i} \left[P_i^*\left(u_{i,j}\right) - P_{i,j}\right]^2 i = 1, 2, \ldots, n$$

Both $P_i\left(u_{i,j}\right)$ and $P_i^*\left(u_{i,j}\right)$ are defined in (1) and (2).

This proposed scheme is more appropriate for surface intersections because calculated error is less than 0.001. This error is minimized by taking comparison with others in literature [10, 11].

3.3 Algorithm

The proposed approach is summarized in the form of an algorithm.

1. Input the data of surfaces.
2. Calculate the required characteristics points from intersection of two surfaces.
3. Find the finest optimal value for shape parameter r_i by GA.
4. Fit the rational quadratic spline for curve fitting. If curve achieved is optimal then go to step 5 else go to step 3 and repeat it until best approximated curve would be achieved.
5. Stop.

4. Demonstrations

The proposed curve fitting scheme, in Section 3, has been applied on explicit-explicit, implicit-explicit, parametric-parametric surface intersection. Algorithm discussed has also been used for getting approximated results. Examples 4.1, 4.2 and 4.3 show the accuracy and efficiency of proposed technique.

Example 4.1. *Two explicit surfaces are given by: (see Figures 1–8)*

$$Es_1 : z = x^2 + y^2 - 1$$

$$Es_2 : z = 2y - x^2$$

Figures 1 and 2 show the explicit surfaces. The first step in proposed scheme is to find the intersection of surfaces which are demonstrated in Figure 3 and also calculate the characteristics points of intersection, shown in Figure 4. Figure 5 represents "sequence of points" of explicit-explicit surface intersection. Figures 6 & 7 depicts 1st *and 2*nd *iterations of GA which are used to give best optimized values to shape parameter, given in rational quadratic spline. Finest curve at 23*rd *iteration of GA is demonstrated in Figure 8.*

Figure 2: Explicit surface-2

Figure 3: Surfaces Intersection

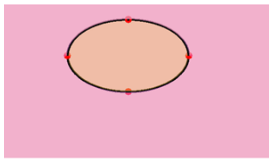

Figure 4: Characteristics points with xy-view of intersection

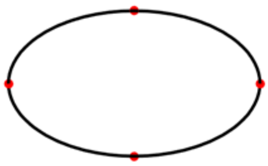

Figure 5: Classification of points

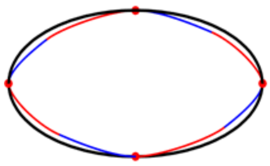

Figure 6: Curve fitted for Ist iteration of GA by rational quadratic spline

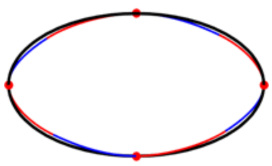

Figure 7: Curve fitted for 2nd iteration of GA by rational quadratic spline

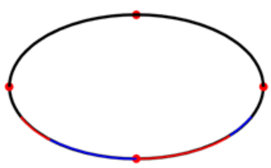

Figure 8: Optimal curve achieved

Figure 9: Implicit surface

Figure 10: Explicit surface

Example 4.2. *Implicit and explicit surfaces are given by: (see Figures 9–16)*

$$Is_1 : x^2 + x^2y^2 - y^2 + z^2 = 1$$

$$Is_2 : z = -x^2 - y^2 + 2$$

Implicit and Explicit surfaces are displayed in Figures 9 and 10. The intersection of surfaces are demonstrated in Figure 11 and the characteristics points of intersection, shown in Figure 12. Figure 13 exemplifies "sequence of points" of implicit-explicit surface intersection. Figures 14 & 15 show 1^{st} and 2^{nd} iterations of GA with curve fitted by rational quadratic spline. Finest curve at 30^{th} iteration of GA is depicted in Figure 16.

Example 4.3. *Two parametric surfaces are given by: (see Figures 17–24)*

$$s_1 = \{(3x, 3y, -6x^2 + 6x - 1) : 0 \leq x, y \leq 1\}$$

$$s_2 = \{(\frac{9}{2}s - \frac{1}{2}, \frac{9}{2}t - \frac{1}{2}, -9st(s-1)(t-1)) : 0 \leq s, t \leq 1\}$$

Parametric surfaces are illustrated in Figures 17 & 18. Intersection of surfaces is given in Figure 19 and characteristics points of intersection, displayed in Figure 20. Figure 21 demonstrates the sequence of points of parametric-parametric surface intersection. Figures 22 & 23 depict 1^{st} and 2^{nd} iterations of GA with curve fitted by rational quadratic spline. Optimal curve at 35^{th} iteration of GA is shown in Figure 24.

5. Conclusion

A Surface to Surface intersection technique is presented in this paper. It detects required characteristics points and approximate them. Genetic Algorithm is used for optimization, which picks up desired optimal values of parameters, in description of rational quadratic. Conics and GA both are used for curve fitting. At the end, the proposed algorithm has reached its target of getting optimal curve with error (<0.001) as compared to [10, 11]. Important thing is concluded that this scheme gives more suitable results with comparison to [6], needs small number of data points which is actually difficult in curve fitting.

Figure 11: Surfaces Intersection

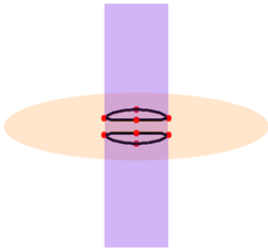

Figure 12: Characteristics points with xy-view of intersection

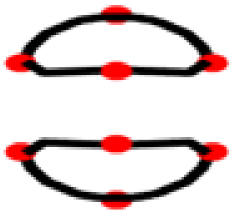

Figure 13: Classification of points

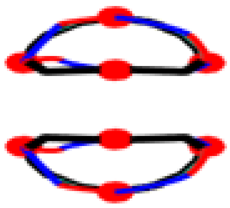

Figure 14: Curve fitted for Ist iteration of GA by rational quadratic spline

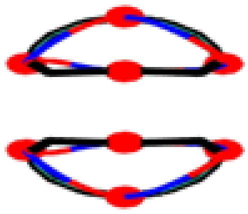

Figure 15: Curve fitted for 2nd iteration of GA by rational quadratic spline

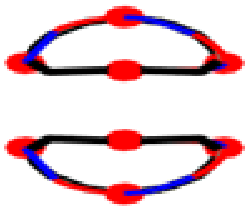

Figure 16: Optimal curve achieved

Figure 17: Parametric surface-1

Figure 18: Parametric surface-2

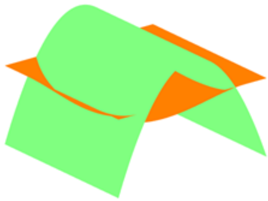

Figure 19: Surfaces Intersection

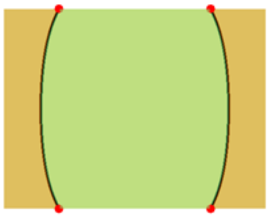

Figure 20: Characteristics points with xy-view of intersection

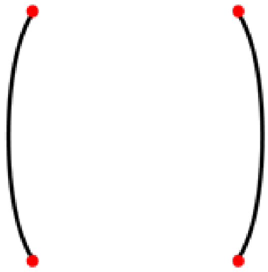

Figure 21: Classification of points

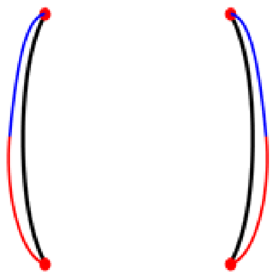

Figure 22: Curve fitted for Ist iteration of GA by rational quadratic spline

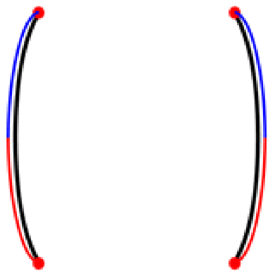

Figure 23: Curve fitted for 2nd iteration of GA by rational quadratic spline

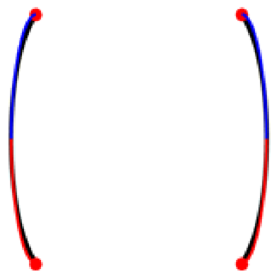

Figure 24: Optimal curve achieved

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