Journal of Corrosion and Materials
www.corromat.com

J. Corros. Mater. **48** (2023) 84–100
https://doi.org/10.61336/jcm2023-10

Investigation of Topological properties of Butterfly Networks by Irregularity Invariants

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(Received: 12 March 2023. Received in revised form: 20 November 2023. Accepted: 27 November 2023. Published online: 02 December 2023.)

Abstract

In the modern era, networks play a crucial role in connecting individuals, organizations, and devices, enabling efficient communication, collaboration, and information exchange across the globe. They serve as the backbone of the digital age, supporting the growth of technologies such as the internet, cloud computing, IoT, and social media. The examination of Quantitative Structure Activity/Property Relationships (QSPR) to analyse the network architecture uses topological indices as its main instrument. The butterfly network is a type of interconnection network that follows a specific geometric structure. It consists of multiple stages, with each stage connecting nodes in a binary pattern. The connections between nodes form a butterfly-like shape, hence the name. The butterfly network provides efficient routing and communication paths between nodes, making it suitable for parallel computing and distributed systems. With the aid of sixteen irregularity indices, we will examine and compare three butterfly networks in this article. We will then plot the findings to see how they depend on the various parameters.

Keywords: Topological indices, Irregularity invariants, Butterfly networks.

1. Introduction

Processing nodes in network interfaces are multiprocessors that are utilised to create a network of identical processor memory pairs. Compiling and running programmes via message sending. High efficiency, more efficient microprocessors and chips have a significant role in the design and use of multiprocessor interconnection networks [1–3]. Interconnection networks were more essential and significant since they approximated the communication pattern of a natural scenario. The vast bulk of networks are interconnected and reliant on one another, and this needs to be taken into account for future development.

Especially, the collaboration that depends on these kind of networks and experiences frequent failures needs to be studied for a better way to address their issues and make enhancements [4, 5]. Buttery Network A popular and significant topological structure of interconnection networks is BF[n]. A butterfly network in graph theory is a structured network topology used in parallel computing. It consists of $log2(N)$ levels, where N is the number of nodes or processors. At each level, there are 2^k nodes, where k is the level number. Nodes at one level are connected to exactly two nodes at the next level, forming a binary tree-like structure. Butterfly networks facilitate efficient communication between nodes in parallel processing systems, making them valuable in distributed computing and interconnection networks. Fast Fourier Transform (FFT), which is frequently employed in the phenomena of signal processing, is achieved by its extensive use for parallel processing and for establishing methodology to analyze it [6, 7]. Numerous suggested architectures for the swapping fabric of air tight high-speed ATM networks make utilization of the butterfly network and other closely related multiphase interconnection networks. By integrating the moderate loops of two butterfly networks, a Benes network is created from the butterfly network [8].

In mathematical chemistry, graph theory offers the trusty tool that is used to compute the numerous types of networks and anticipate their various attributes. A topological index is one of the most crucial tools in chemical graph theory, which is useful in predicting the chemical and physical properties of underlying structure, like strain energy, rigidity, heat of evaporation, tension and so on. [9, 10]. A simple graph is one without several loops or edges. A molecular graph is a fundamental graph in which the vertex and edge sets represent, respectively, atoms and bounds [11, 12]. The count of edges that are attached to a vertex determines its degree. It is mostly interesting these characteristics of diverse objects. Winner discovered the boiling point in 1947 [13] and simultaneously introduced the idea of the first topological index. Gutman provided a noteworthy identity regarding Zagreb indices in 1975 [14]. As a result, these two indices are some of the first degree-based descriptors, and substantial research has been done on their characteristics. If $\mathbb{G}$ be a graph and α and β are the vertices of an edge. Then the mathematical formulae of above mentioned invariants are:

$$M_1(\mathbb{G}) = \sum_{\alpha\beta \in E(\mathbb{G})} (d_\alpha + d_\beta)$$

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$$M_2(\mathbb{G}) = \sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha \times d_\beta)$$

The irregularity topological index is a graph attribute that quantifies the irregularity or heterogeneity of a network's connectivity pattern. It measures the deviation of the network's degrees (number of connections) from their average value [15]. The irregularity topological index is primarily used in network analysis to understand the irregular behavior of network. It provides insights into the structural properties and complexity of networks. Higher values of the index indicate greater irregularity in the network's connectivity, suggesting a more diverse distribution of connections among nodes. This index helps researchers understand the robustness, resilience, and efficiency of networks, making it useful in fields [16–18].

Irregularity Index	**Mathematical Form**
VAR	$\sum_{\alpha\in V(\mathbb{G})} (d_u - \frac{2m}{n})^2 = \frac{M_1(\mathbb{G})}{n} - (\frac{2m}{n})^2$
AL	$\sum_{\alpha\beta\in E(\mathbb{G})} \lvert d_\alpha - d_\beta\rvert$
IR1	$\sum_{\alpha\in V(\mathbb{G})} (d_\alpha)^3 - \frac{2m}{n}\sum_{u\in V} (d_\alpha)^2 = F(\mathbb{G}) - \frac{2m}{n}M_1(\mathbb{G})$
IR2	$\sqrt{\frac{\sum_{uv\in E(\mathbb{G})} d_\alpha d_\beta}{m}} - \frac{2m}{n} = \sqrt{\frac{M_2(\mathbb{G})}{m}} - \frac{2m}{n}$
IRF	$\sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha - d_\beta)^2 = F(\mathbb{G}) - 2M_2(\mathbb{G})$
IRFW	$\frac{IRF(\mathbb{G})}{M_2(\mathbb{G})}$
IRA	$\sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha^{-1/2} - d_\beta^{-1/2})^2 = n - 2R(\mathbb{G})$
IRB	$\sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha^{1/2} - d_\beta^{1/2})^2 = M_1(\mathbb{G}) - 2RR(\mathbb{G})$
IRC	$\frac{\sum_{uv\in E(\mathbb{G})} \sqrt{d_\alpha d_\beta}}{m} - \frac{2m}{n} = \frac{RR(\mathbb{G})}{m} - \frac{2m}{n}$
IRDIF	$\sum_{\alpha\beta\in E(\mathbb{G})} \lvert\frac{d_\alpha}{d_\beta} - \frac{d_\alpha}{d_\beta}\rvert = \sum_{i<j} m_{i,j}(\frac{j}{i} - \frac{i}{j})$
IRL	$\sum_{\alpha\beta\in E(\mathbb{G})} \lvert lnd_\alpha - lnd_\beta\rvert = \sum_{i<j} m_{i,j}ln(\frac{j}{i})$
IRLU	$\sum_{\alpha\beta\in E(\mathbb{G})} \frac{\lvert d_\alpha - d_\beta\rvert}{min(d_u,d_v)} = \sum_{i<j} m_{i,j}ln(\frac{j-i}{i})$
IRLF	$\sum_{\alpha\beta\in E(\mathbb{G})} \frac{\lvert d_\alpha - d_\beta\rvert}{\sqrt{(d_\alpha d_\beta)}} = \sum_{i<j} m_{i,j}(\frac{j-i}{\sqrt{ij}})$
IRLA	$2\sum_{\alpha\beta\in E(\mathbb{G})} \frac{\lvert d_\alpha - d_\beta\rvert}{(d_\alpha + d_\beta)} = 2\sum_{i<j} m_{i,j}(\frac{j-i}{i+j})$
IRDI	$\sum_{\alpha\beta\in E(\mathbb{G})} ln1 + \lvert d_\alpha - d_\beta\rvert = \sum_{i<j} m_{i,j}ln(i+j-1)$
IRGA	$\sum_{\alpha\beta\in E(\mathbb{G})} ln(\frac{d_\alpha + d_\beta}{2\sqrt{d_u d_v}}) \sum_{i<j} m_{i,j}(\frac{i+j}{2\sqrt{ij}})$

2. Irregularity indices for Buttery Networks

The irregularity indices of butter y networks are discussed in this section. Here, we will discuss three different structures of butterfly networks namely butterfly network $BF(t)$, Horizontal Cylindrical Butterfly Network $HCBF(t)$ and Toroidal Buttery Network $TBF(t)$. Firstly associate a graph with butterfly networks and then discuss different butterfly networks by means of irregularity indices.

2.1 Irregularity indices for Buttery Network $BF(t)$

Here, we will investigate sixteen irregularity indices for butterfly network $BF(t)$. The molecular graph of $BF(t)$ is given in Figure 1. The edge division of 3D Butterfly Network is given in Table 1

(d_u, d_v)	Frequency
(2,4)	2^{t+1}
(4,4)	$2^{t+1}(t-2)$

Table 1: $E(BF(t))$

Theorem 2.1. *Let $\mathbb{G}$ be the graph of Butterfly Network $BF(t)$. Then the Irregularity indices are*

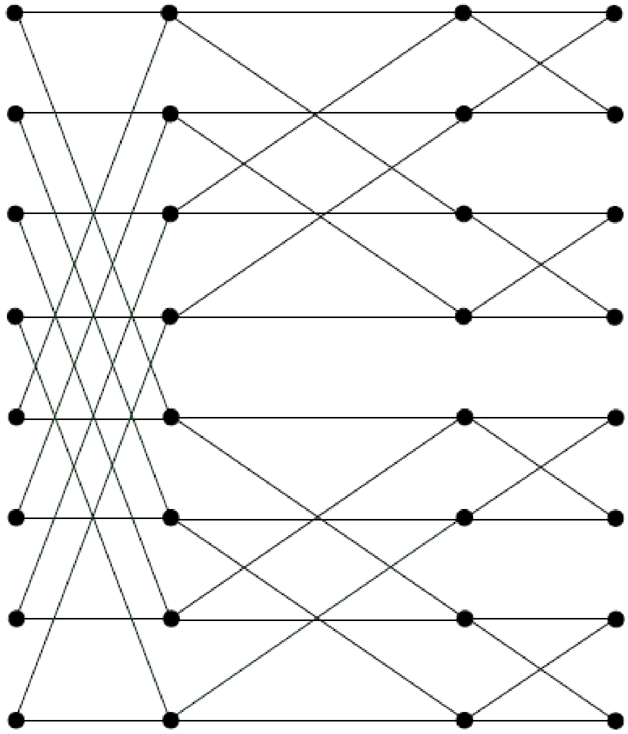

Figure 1: 3D Butterfly Network

1. $VAR(\mathbb{G}) = \frac{8(2t^2-t-5)}{(1+t)^2}$
2. $AL(\mathbb{G}) = 2.2^{t+2}$
3. $IR1(\mathbb{G}) = \frac{16.2^t(7t-11)}{1+t}$
4. $IR2(\mathbb{G}) = 4\sqrt{\frac{2^t(-3+2t)}{t2^{(1+t)}}} - \frac{2^{(2+t)}t}{(1+t)2^t}$
5. $IRF(\mathbb{G}) = (4)2^(t+2)$
6. $IRFW(\mathbb{G}) = \frac{2^{t+2}}{8(2^t(-3+2t))}$
7. $IRA(\mathbb{G}) = 2^t(5-t-2^{3/2})$
8. $IRB(\mathbb{G}) = 2^t(24-16.2^{1/2})$
9. $IRC(\mathbb{G}) = \frac{2(t\sqrt{2}+\sqrt{2}-2t-4)}{t(1+t)}$
10. $IRDIF(\mathbb{G}) = (1.5)2^{t+2}$
11. $IRL(\mathbb{G}) = (0.6931)2^{t+2}$
12. $IRLU(\mathbb{G}) = 2^{t+2}$
13. $IRLF(\mathbb{G}) = (0.7071)2^{t+2}$
14. $IRLA(\mathbb{G}) = (0.6667)2^{t+2}$
15. $IRD1(\mathbb{G}) = (1.0986)2^{t+2}$
16. $IRGA(\mathbb{G}) = (0.0588)2^{t+2}$

Proof.

$$\begin{aligned} VAR(\mathbb{G}) &= \sum_{\alpha\epsilon V}\left(d_\alpha - \frac{2m}{n}\right)^2 = \frac{M_1(\mathbb{G})}{n} - \left(\frac{2m}{n}\right)^2 \\ &= \left(\frac{8.2^t(-5+4t)}{(t+1)2^t}\right) - \left(\frac{2(t2^{t+1})}{(t+1)2^t}\right)^2 \end{aligned}$$

$$= \frac{8(2t^2 - t - 5)}{(1+t)^2}.$$

$$AL(\mathbb{G}) = \sum_{\alpha\beta\epsilon E(\mathbb{G})} |d_\alpha - d_\beta|$$
$$= |2-4|(2^{t+2}) + |4-4|(2^{t+2})(t-2)$$
$$= (2)2^{t+2}.$$

$$IR1(\mathbb{G}) = \sum_{\alpha\epsilon V} d_\alpha^3 - \frac{2m}{n}\sum_{\alpha\epsilon V} d_\alpha^2 = F(\mathbb{G}) - \left(\frac{2m}{n}\right) M_1(\mathbb{G})$$
$$= (16.2^t(-11+8t)) - \left(\frac{2(t2^{t+1})}{(t+1)2^t}\right)(8.2^t(-5+4t))$$
$$= \frac{16.2^t(7t-11)}{1+t}.$$

$$IR2(\mathbb{G}) = \sqrt{\frac{\sum_{\alpha\beta\in E(\mathbb{G})} d_\alpha d_\beta}{m}} - \frac{2m}{n} = \sqrt{\frac{M_2(\mathbb{G})}{m}} - \frac{2m}{n}$$
$$= \sqrt{\frac{32.2^t(-3+2t)}{(t+1)2^t}} - \left(\frac{2(t2^{t+1})}{(t+1)2^t}\right)$$
$$= 4\sqrt{\frac{2^t(-3+2t)}{t2^{(1+t)}}} - \frac{2^{2+t})t}{(1+t)2^t}.$$

$$IRF(\mathbb{G}) = \sum_{\alpha\beta\epsilon E(\mathbb{G})} (d_\alpha - d_\beta)^2$$
$$= (2-4)^2(2^{t+2}) + (4-4)^2(2^{t+2})(t-2)$$
$$= (4)2^(t+2).$$

$$IRFW(\mathbb{G}) = \frac{IRF(\mathbb{G})}{M_2(\mathbb{G})}$$
$$= \frac{2^{t+2}}{8(2^t(-3+2t))}.$$

$$IRA(\mathbb{G}) = \sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha^{-1/2} - d_\beta^{-1/2})^2 = n - 2R(\mathbb{G})$$
$$= (t+1)2^t - \frac{1}{2}(\sqrt{2}2^{(t+2)} + 2^{(t+2)}(t-2))$$
$$= 2^t(5 - t - 2^{3/2}).$$

$$IRB(\mathbb{G}) = \sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha^{1/2} - d_\beta^{1/2})^2 = M_1(\mathbb{G}) - 2RR(\mathbb{G})$$
$$= (8.2^t(-5+4t)) - 2(2\sqrt{2}2^{(t+2)} + 4.2^{(t+2)}(t-2))$$
$$= 2^t(24 - 16.2^{1/2}).$$

$$IRC(\mathbb{G}) = \frac{\sum_{\alpha\beta\in E(\mathbb{G})} \sqrt{d_\alpha d_\beta}}{m} - \frac{2m}{n} = \frac{RR(\mathbb{G})}{m} - \frac{2m}{n}$$
$$= \frac{\frac{1}{4}(2\sqrt{2}2^{(t+2)} + 4.2^{(t+2)}(t-2)}{(t+1)2^t} - \left(\frac{2(t2^{t+1})}{(t+1)2^t}\right)$$
$$= \frac{2(t\sqrt{2} + \sqrt{2} - 2t - 4)}{t(1+t)}.$$

$$IRDIF(\mathbb{G}) = \sum_{\alpha\beta\in E(\mathbb{G})} |\frac{d_\alpha}{d_\beta} - \frac{d_\alpha}{d_\beta}|$$

$$
\begin{aligned}
&= |\frac{2}{4}-\frac{4}{2}|(2^{t+2})+|\frac{4}{4}-\frac{4}{4}|(2^{t+2})(t-2)\\
&= (1.5)2^{t+2}.
\end{aligned}
$$

$$
\begin{aligned}
IRL(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} |lnd_\alpha - lnd_\beta|\\
&= |ln2-ln4|(2^{t+2})+|ln4-ln4|(2^{t+2})(t-2)\\
&= (0.6931)2^{t+2}.
\end{aligned}
$$

$$
\begin{aligned}
IRLU(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} \frac{|d_\alpha - d_\beta|}{min(d_\alpha, d_\beta)}\\
&= \frac{|2-4|}{2}(2^{t+2})+\frac{|4-4|}{4}(2^{t+2})(t-2)\\
&= 2^{t+2}.
\end{aligned}
$$

$$
\begin{aligned}
IRLF(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} \frac{|d_\alpha - d_\beta|}{\sqrt{d_\alpha.d_\beta}}\\
&= \frac{|2-4|}{\sqrt{6}}(2^{t+2})+\frac{|4-4|}{\sqrt{16}}4(2^{t+2})(t-2)\\
&= (0.7071)2^{t+2}.
\end{aligned}
$$

$$
\begin{aligned}
IRLA(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} 2\frac{|d_\alpha - d_\beta|}{(d_\alpha + d_\beta)}\\
&= 2\frac{|2-4|}{6}((2^{t+2}))+2\frac{|4-4|}{8}(2^{t+2})(t-2)\\
&= (0.6667)2^{t+2}.
\end{aligned}
$$

$$
\begin{aligned}
IRD1(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} ln\{1+|d_\alpha - d_\beta|\}\\
&= ln\{1+|2-4|\}((2^{t+2}))+ln\{1+|4-4|\}(2^{t+2})(t-2)\\
&= (1.0986)2^{t+2}.
\end{aligned}
$$

$$
\begin{aligned}
IRGA(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} ln\left(\frac{d_\alpha + d_\beta}{2\sqrt{d_\alpha d_\beta}}\right)\\
&= ln\left(\frac{2+4}{2\sqrt{2\times 4}}\right)((2^{t+2}))+ln\left(\frac{4+4}{2\sqrt{4\times 4}}\right)(2^{t+2})(t-2)\\
&= (0.0588)2^{t+2}.
\end{aligned}
$$

□

2.2 Irregularity indices for Buttery Network Cylindrical representation of Butterfly Network(horizontal identification) $HCBF(t)$

In this section, we will discuss the irregularity indices for Buttery Network Cylindrical representation of Butterfly Network(horizontal identification) $HCBF(t)$. The molecular graph of $HCBF$ is given in Figure 2. The edge partition of 3D Horizontal Cylindrical Butterfly Network is given in Table 3.

Theorem 2.2. *Let $\mathbb{G}$ be the graph of Butterfly network $HCBF(t)$. Then the Irregularity indices are*

1. $VAR(\mathbb{G}) = \frac{-42^t t^2 - 84^t t + 14t2^t + 8t^2 + 2^{t+1} + 84^t - 6t - 10}{(t+1)^2(2^t-1)^2}$
2. $AL(\mathbb{G}) = 8t + 2.2^{t+2} - 14$
3. $IR1(\mathbb{G}) = \frac{2(20.2^t t^2 + 24.4^t t - 31t2^t - 40t^2 - 24.4^t - 39.2^t + 17t + 63)}{(t+1)(2^t-1)}$

	t=1	t=2	t=3	t=4	t=5
VAR	-8	0.88	5	7.36	8.88
AL	16	32	64	128	256
IR1	-64	64	320	870.4	2048
IR2	-	0.66	-0.17	-0.04	0.04
IRF	32	64	128	256	512
IRFW	-0.50	0.50	0.16	0.10	0.07
IRA	2.37	-1.85	-6.62	-29.25	-90.50
IRB	2.75	5.49	10.98	21.96	43.92
IRC	3.17	1.24	2.1	-0.5	0.34
IRDIF	12	24	48	96	192
IRL	5.54	11.08	22.17	44.35	88.71
IRLU	8	16	32	64	128
IRLF	12	24	48	96	192
IRLA	5.33	10.67	21.34	42.68	85.37
IRD1	8.78	17.57	35.13	70.31	140.62
IRGA	047	0.94	1.88	3.76	7.52

Table 2: Irregularity invariants for Butterfly network $BF(t)$ for different values of t

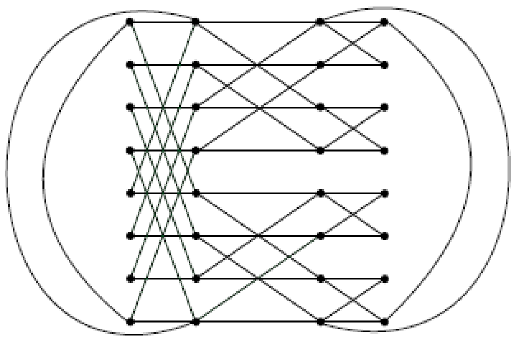

Figure 2: 3D Horizontal Cylindrical Butterfly Network

4. $IR2(\mathbb{G}) = \sqrt{\frac{32t2^t - 322^t + 36t - 36}{(t+1)(2^t-1)}} - \frac{2t(2^(t+1)-1)}{(t+1)(2^t-1)}$

5. $IRF(\mathbb{G}) = 16t + 4.2^{t+2} + 6$

6. $IRFW(\mathbb{G}) = \frac{16t + 4.2^{t+2} + 6}{32t2^t - 32.2^t + 36t - 36}$

7. $IRA(\mathbb{G}) = \frac{1}{3}(9.2^t + 5t - 7 - 4\sqrt{3} + 16\sqrt{(2)} - 2\sqrt{6}t + 4\sqrt{6} - 3.2^{3/2+t})$

8. $IRB(\mathbb{G}) = -16\sqrt{6}t + 32\sqrt{6} - 16.2^{1/2+t} + 24.2^t - 32\sqrt{(3)} + 36\sqrt{(2)} + 40t - 82$

9. $IRC(\mathbb{G}) = \frac{1}{t(2^{t+1}-1)(t+1)(2^t-1)}(2(-4.2^{(1/2+t)}\sqrt{3}t^2 + 4\sqrt{6}t^2 + 4.2^{(1/2+t)}\sqrt{3}t - 4\sqrt{6}t - 8.\sqrt{3}2^t t + 9.2^t t^2 + 2^{2t+2}t - 2^{(5/2+2t)}t + 8.2^{(1/2+t)}\sqrt{3} + 13.2^{(1/2+t)}t - 8\sqrt{6} - 9\sqrt{2}t - 8\sqrt{3}2^t + 8\sqrt{3}t - 13t2^t - 8t^2 + 8.4^t - 2^{5/2+2t} + 13.2^{(1/2+t)} - 9\sqrt{2} + 8\sqrt{3} - 26.2^t + 9t + 18))$

10. $IRDIF(\mathbb{G}) = (1.5)2^{(t+2)} + (3.3332)t - (8.6664)$

11. $IRL(\mathbb{G}) = (0.6931)2^{(t+2)} + (1.6216)t - (4.6294)$

12. $IRLU(\mathbb{G}) = 2^{(t+2)} + 2t - (4.6668)$

13. $IRLF(\mathbb{G}) = (0.7071)2^{(t+2)} + (1.6328)t - (6.8730)$

14. $IRLA(\mathbb{G}) = (0.6667)2^{(t+2)} + (1.6)t - (7.1242)$

15. $IRD1(\mathbb{G}) = (1.0986)2^{(t+2)} + (4.3944)t - (12.0348)$

16. $IRGA(\mathbb{G}) = (0.0588)2^{(t+2)} + (0.0816)t - (0.1348)$

(d_u, d_v)	Frequency
(2,6)	4
(3,6)	2
(6,6)	$t-2$
(4,6)	$4(t-2)$
(3,4)	4
(4,2)	$2^{t+2}-12$
(4,4)	$(t-2)(2^{t+1}-6)$

Table 3: $E(HCBF(t))$

Proof.

$$\begin{aligned} VAR(\mathbb{G}) &= \sum_{\alpha\epsilon V}\left(d_\alpha - \frac{2m}{n}\right)^2 = \frac{M_1(\mathbb{G})}{n} - \left(\frac{2m}{n}\right)^2 \\ &= \left(\frac{16.t2^t - 8.2^t + 4t - 10}{(t+1)(2^t-1)}\right) - \left(\frac{2t(2t+1-1)}{(t+1)(2^t-1)}\right)^2 \\ &= \frac{-42^t t^2 - 84^t t + 14t2^t + 8t^2 + 2^{t+1} + 84^t - 6t - 10}{(t+1)^2(2^t-1)^2}. \end{aligned}$$

$$\begin{aligned} AL(\mathbb{G}) &= \sum_{uv\epsilon E(\mathbb{G})} |d_\alpha - d_\beta| \\ &= |2-6|(4) + |3-6|(2) + |6-6|(t-2) + |4-6|(4t-8) \\ &\quad +|3-4|(4) + |4-2|(2^{t+2}-12) + |4-4|(t-2)(2^{t+1}-6) \\ &= 8t + 2.2^{t+2} - 14. \end{aligned}$$

$$\begin{aligned} IR1(\mathbb{G}) &= \sum_{\alpha\epsilon V} d_\alpha^3 - \frac{2m}{n}\sum_{\alpha\epsilon V} d_\alpha^2 = F(\mathbb{G}) - \left(\frac{2m}{n}\right) M_1(\mathbb{G}) \\ &= (64t2^t - 48.2^t + 88t - 126) - \left(\frac{2t(2t+1-1)}{(t+1)(2^t-1)}\right)(16t2^t - 8.2^t + 4t - 10) \\ &= \frac{2(20.2^t t^2 + 24.4^t t - 31t2^t - 40t^2 - 24.4^t - 39.2^t + 17t + 63)}{(t+1)(2^t-1)}. \end{aligned}$$

$$\begin{aligned} IR2(\mathbb{G}) &= \sqrt{\frac{\sum_{\alpha\beta\in E(\mathbb{G})} d_\alpha d_\beta}{m}} - \frac{2m}{n} = \sqrt{\frac{M_2(\mathbb{G})}{m}} - \frac{2m}{n} \\ &= \sqrt{\frac{32t2^t - 32.2^t + 36t - 36}{t(2t+1-1)}} - \left(\frac{2t(2t+1-1)}{(t+1)(2^t-1)}\right) \\ &= \sqrt{\frac{32t2^t - 322^t + 36t - 36}{(t+1)(2^t-1)}} - \frac{2t(2^(t+1)-1)}{(t+1)(2^t-1)}. \end{aligned}$$

$$\begin{aligned} IRF(\mathbb{G}) &= \sum_{uv\epsilon E(\mathbb{G})} (d_\alpha - d_\beta)^2 \\ &= (2-6)^2(4) + (3-6)^2(2) + (6-6)^2(t-2) + (4-6)^2(4t-8) \\ &\quad +(3-4)^2(4) + (4-2)^2(2^{t+2}-12) + (4-4)^2(t-2)(2^{t+1}-6) \\ &= 16t + 4.2^{t+2} + 6. \end{aligned}$$

$$\begin{aligned} IRFW(\mathbb{G}) &= \frac{IRF(\mathbb{G})}{M_2(\mathbb{G})} \\ &= \frac{16t + 4.2^{t+2} + 6}{32t2^t - 32.2^t + 36t - 36}. \end{aligned}$$

$$IRA(\mathbb{G}) = \sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha^{-1/2} - d_\beta^{-1/2})^2 = n - 2R(\mathbb{G})$$

$$
\begin{aligned}
&= (t+1)(2^t-1)-2((\frac{2}{3}\sqrt{3}+\frac{1}{3}\sqrt{2}+\frac{1}{6}t \\
&\quad +\frac{1}{12}\sqrt{6}(4t-8)+\frac{1}{4}\sqrt{2}(2^{t+}-12)+(\frac{1}{4}(t-2))(2^{(t+1)}-6)) \\
&= \frac{1}{3}(9.2^t+5t-7-4\sqrt{3}+16\sqrt{(2)}-2\sqrt{6}t+4\sqrt{6}-3.2^{3/2+t}).
\end{aligned}
$$

$$
\begin{aligned}
IRB(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})}(d_\alpha^{1/2}-d_\beta^{1/2})^2 = M_1(\mathbb{G})-2RR(\mathbb{G}) \\
&= (16t2^t-8.2^t+4t-10)-2((4t-8)2^{(t+1)} \\
&\quad +(2.2^{(t+2)}-18)\sqrt{2}+(8t-16)\sqrt{6}-18t+16\sqrt{3}+36) \\
&= -16\sqrt{6}t+32\sqrt{6}-16.2^{1/2+t}+24.2^t-32\sqrt{(3)}+36\sqrt{(2)}+40t-82.
\end{aligned}
$$

$$
\begin{aligned}
IRC(\mathbb{G}) &= \frac{\sum_{\alpha\beta\in E(\mathbb{G})}\sqrt{d_\alpha d_\beta}}{m}-\frac{2m}{n} = \frac{RR(\mathbb{G})}{m}-\frac{2m}{n} \\
&= \frac{((4t-8)2^{(t+1)}+(2.2^{(t+2)}-18)\sqrt{2}+(8t-16)\sqrt{6}-18t+16\sqrt{3}+36)}{t(2^{t+1}-1)} \\
&\quad -\left(\frac{2t(2t+1-1)}{(t+1)(2^t-1)}\right) \\
&= \frac{1}{t(2^{t+1}-1)(t+1)(2^t-1)}(2(-4.2^{(1/2+t)}\sqrt{3}t^2+4\sqrt{6}t^2+ \\
&\quad 4.2^{(1/2+t)}\sqrt{3}t-4\sqrt{6}t-8.\sqrt{3}2^t t+9.2^t t^2+2^{2t+2}t-2^{(5/2+2t)}t+8.2^{(1/2+t)}\sqrt{3} \\
&\quad +13.2^{(1/2+t)}t-8\sqrt{6}-9\sqrt{2}t-8\sqrt{3}2^t+8\sqrt{3}t-13t2^t-8t^2+8.4^t-2^{5/2+2t} \\
&\quad +13.2^{(1/2+t)}-9\sqrt{2}+8\sqrt{3}-26.2^t+9t+18)).
\end{aligned}
$$

$$
\begin{aligned}
IRDIF(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})}|\frac{d_\alpha}{d_\beta}-\frac{d_\alpha}{d_\beta}| \\
&= |\frac{2}{6}-\frac{6}{2}|(4)+|\frac{3}{6}-\frac{6}{3}|(2) \\
&\quad +|\frac{4}{4}-\frac{4}{4}|(t-2)+|\frac{4}{6}-\frac{6}{4}|(4t-8)+|\frac{4}{3}-\frac{3}{4}|(4) \\
&\quad +|\frac{4}{2}-\frac{4}{2}|(2^{t+2}-12) \\
&\quad +|\frac{4}{4}-\frac{4}{4}|(t-2)(2^{t+1}-6) \\
&= (1.5)2^{(t+2)}+(3.3332)t-(8.6664).
\end{aligned}
$$

$$
\begin{aligned}
IRL(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})}|lnd_\alpha-lnd_\beta| \\
&= |ln2-ln6|(4)+|ln3-ln6|(2)+|ln6-ln6|(t-2)+|ln4-ln6|(4t-8) \\
&\quad +|ln3-ln4|(4)+|ln4-ln2|(2^{t+2}-12)+|ln4-ln4|(t-2)(2^{t+1}-6) \\
&= (0.6931)2^{(t+2)}+(1.6216)t-(4.6294).
\end{aligned}
$$

$$
\begin{aligned}
IRLU(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})}\frac{|d_\alpha-d_\beta|}{min(d_\alpha,d_\beta)} \\
&= \frac{|2-6|}{2}(4)+\frac{|3-6|}{3}(2)+\frac{|6-6|}{6}(t-2)+\frac{|4-6|}{4}(4t-8) \\
&\quad +\frac{|3-4|}{3}(4)+\frac{|4-2|}{2}(2^{t+2}-12)+\frac{|4-4|}{4}(t-2)(2^{t+1}-6) \\
&= 2^{(t+2)}+2t-(4.6668).
\end{aligned}
$$

$$
\begin{aligned}
IRLF(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})}\frac{|d_\alpha-d_\beta|}{\sqrt{d_\alpha.d_\beta}} \\
&= \frac{|2-6|}{\sqrt{12}}(4)+\frac{|3-6|}{\sqrt{18}}(2)+\frac{|6-6|}{\sqrt{36}}(t-2)+\frac{|4-6|}{\sqrt{24}}(4t-8)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{|3-4|}{\sqrt{12}}(4)+\frac{|4-2|}{\sqrt{8}}(2^{t+2}-12)+\frac{|4-4|}{\sqrt{16}}(t-2)(2^{t+1}-6) \\
= & (0.7071)2^{(t+2)}+(1.6328)t-(6.8730).
\end{aligned}
$$

$$
\begin{aligned}
IRLA(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} 2\frac{|d_\alpha-d_\beta|}{(d_\alpha+d_\beta)} \\
&= 2\frac{|2-6|}{8}(4)+2\frac{|3-6|}{9}(2)+2\frac{|6-6|}{12}(t-2)+2\frac{|4-6|}{24}(4t-8) \\
&\quad +2\frac{|3-4|}{7}(4)+2\frac{|4-2|}{6}(2^{t+2}-12)+2\frac{|4-4|}{8}(t-2)(2^{t+1}-6) \\
&= (0.6667)2^{(t+2)}+(1.6)t-(7.1242).
\end{aligned}
$$

$$
\begin{aligned}
IRD1(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} ln\{1+|d_\alpha-d_\beta|\} \\
&= ln\{1+|2-6|\}(4)+ln\{1+|3-6|\}(2)+ln\{1+|6-6|\}(t-2) \\
&\quad +ln\{1+|4-6|\}24(4t-8)+ln\{1+|3-4|\}(4)+ln\{1+|4-2|\}(2^{t+2}-12) \\
&\quad +ln\{1+|4-4|\}(t-2)(2^{t+1}-6) \\
&= (1.0986)2^{(t+2)}+(4.3944)t-(12.0348).
\end{aligned}
$$

$$
\begin{aligned}
IRGA(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} ln\left(\frac{d_\alpha+d_\beta}{2\sqrt{d_\alpha d_\beta}}\right) \\
&= ln\left(\frac{2+6}{2\sqrt{2\times 6}}\right)(4)+ln\left(\frac{3+6}{2\sqrt{3\times 6}}\right)(2)+ln\left(\frac{6+6}{2\sqrt{6\times 6}}\right)(t-2) \\
&\quad +ln\left(\frac{4+6}{2\sqrt{4\times 6}}\right)24(4t-8)+ln\left(\frac{3+4}{2\sqrt{3\times 4}}\right)(4)+ln\left(\frac{4+2}{2\sqrt{4\times 2}}\right)(2^{t+2}-12) \\
&\quad +ln\left(\frac{4+4}{2\sqrt{4\times 4}}\right)(t-2)(2^{t+1}-6) \\
&= (0.0588)2^{(t+2)}+(0.0816)t-(0.1348).
\end{aligned}
$$

□

	t=1	t=2	t=3	t=4	t=5
VAR	-4	0.76	0.14	1.09	0.96
AL	10	34	74	146	282
IR1	-60	52.66	229.28	544.93	1153.16
IR2	-3	1.11	1.27	1.38	1.51
IRF	54	102	182	326	598
IRFW	-	2.12	2.33	3.38	5.45
IRA	1.87	0.58	2.97	4.37	7.15
IRB	6.44	2.26	3.04	4.49	7.41
IRC	-0.57	0.31	0.29	0.25	0.20
IRDIF	6.66	22	49.33	100.66	199.99
IRL	2.53	9.70	22.41	46.21	92.19
IRLU	5.33	15.33	33.33	67.33	133.3
IRLF	0.42	7.70	20.65	44.91	91.79
IRLA	-0.19	6.74	19.01	41.94	86.21
IRD1	1.15	14.33	36.30	75.85	150.56
IRGA	0.41	0.96	1.99	3.95	7.79

Table 4: Irregularity invariants for Butterfly network $HCBF(t)$ for different values of t

2.3 Irregularity indices for Toroidal Buttery Network $TBF(t)$

In this section, we will compute irregularity indices for Toroidal Buttery Network $TBF(t)$. The molecular graph of Toroidal Buttery Network $TBF(t)$ is given in 3. The edge partition of Toroidal Buttery Network $TBF(t)$ is given in Table 5

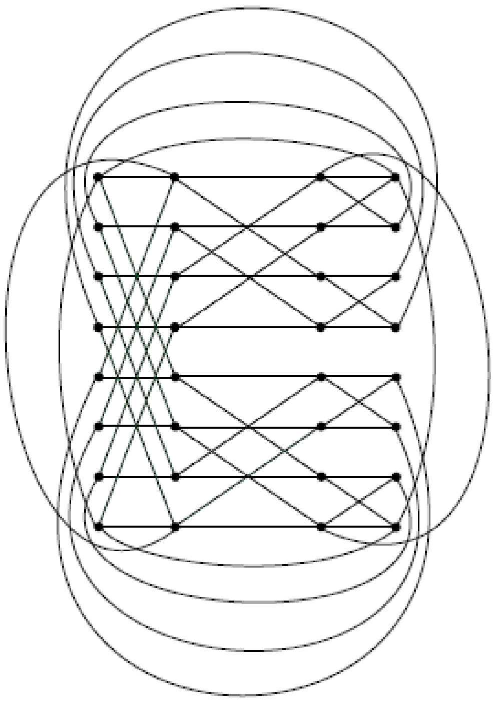

Figure 3: Toroidal Buttery Network $TBF(t)$

(d_u, d_v)	Frequency
(4,6)	$t(2^{t+1}-1)$

Table 5: $E(TBF(t))$

Theorem 2.3. *Let $\mathbb{G}$ be the graph of Butterfly network $TBF(t)$. Then the Irregularity indices are*

1. $VAR(\mathbb{G}) = \frac{2(2.4^t-7.2^t+3)}{(2^t-1)^2}$
2. $AL(\mathbb{G}) = 2t(2^{t+1}-1)$
3. $IR1(\mathbb{G}) = \frac{4t(6.4^t-19.2^t+8)}{2^t-1}$
4. $IR2(\mathbb{G}) = \frac{2^{t+1}\sqrt{6}-2\sqrt{6}-4.2^t+2}{2^t-1}$
5. $IRF(\mathbb{G}) = 4t(2^{t+2}-1)$
6. $IRFW(\mathbb{G}) = \frac{1}{12}$
7. $IRA(\mathbb{G}) = \frac{1}{6}t(6.2^t-2\sqrt{6}2^t+\sqrt{6}-6)$
8. $IRB(\mathbb{G}) = (10t-4\sqrt{6}t)(2^{t+1}-1)$
9. $IRC(\mathbb{G}) = \frac{4.2^t+2\sqrt{6}-\sqrt{6}2^{t+1}-2}{2^t-1}$
10. $IRDIF(\mathbb{G}) = (0.8333)t(2^{t+1}-1)$
11. $IRL(\mathbb{G}) = (0.4054)t(2^{t+1}-1)$
12. $IRLU(\mathbb{G}) = (0.5)t(2^{t+1}-1)$
13. $IRLF(\mathbb{G}) = (0.4082)t(2^{t+1}-1)$
14. $IRLA(\mathbb{G}) = (0.4)t(2^{t+1}-1)$
15. $IRD1(\mathbb{G}) = (1.0986)t(2^{t+1}-1)$
16. $IRGA(\mathbb{G}) = (0.0204)t(2^{t+1}-1)$

Proof.

$$\begin{aligned} VAR(\mathbb{G}) &= \sum_{\alpha\epsilon V}\left(d_\alpha - \frac{2m}{n}\right)^2 = \frac{M_1(\mathbb{G})}{n} - \left(\frac{2m}{n}\right)^2 \\ &= \left(\frac{10t(2^{(t+1)}-1)}{t(2^t-1)}\right) - \left(\frac{2(t(2^{t+1}-1))}{t(2^t-1)}\right)^2 \\ &= \frac{2(2.4^t - 7.2^t + 3)}{(2^t-1)^2}. \end{aligned}$$

$$\begin{aligned} AL(\mathbb{G}) &= \sum_{uv\epsilon E(\mathbb{G})} |d_\alpha - d_\beta| \\ &= |4-6|t(2^{t+1}-1) \\ &= 2t(2^{t+1}-1). \end{aligned}$$

$$\begin{aligned} IR1(\mathbb{G}) &= \sum_{\alpha\in V} d_\alpha^3 - \frac{2m}{n}\sum_{\alpha\epsilon V} d_\alpha^2 = F(\mathbb{G}) - \left(\frac{2m}{n}\right) M_1(\mathbb{G}) \\ &= (52t(2^{(t+1)}-1)) - \left(\frac{2(t(2^{t+1}-1))}{t(2^t-1)}\right)(10t(2^{(t+1)}-1)) \\ &= \frac{4t(6.4^t - 19.2^t + 8)}{2^t-1}. \end{aligned}$$

$$\begin{aligned} IR2(\mathbb{G}) &= \sqrt{\frac{\sum_{\alpha\beta\in E(\mathbb{G})} d_\alpha d_\beta}{m}} - \frac{2m}{n} = \sqrt{\frac{M_2(\mathbb{G})}{m}} - \frac{2m}{n} \\ &= \sqrt{\frac{(24t(2^{(t+1)}-1))}{t(2^t-1)}} - \left(\frac{2(t(2^{t+1}-1))}{t(2^t-1)}\right) \\ &= \frac{2^{t+1}\sqrt{6} - 2\sqrt{6} - 4.2^t + 2}{2^t-1}. \end{aligned}$$

$$\begin{aligned} IRF(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha - d_\beta)^2 \\ &= (4-6)^2 t(2^{t+1}-1) \\ &= 4t(2^{t+1}-1). \end{aligned}$$

$$\begin{aligned} IRFW(\mathbb{G}) &= \frac{IRF(\mathbb{G})}{M_2(\mathbb{G})} \\ &= \frac{1}{12}. \end{aligned}$$

$$\begin{aligned} IRA(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha^{-1/2} - d_\beta^{-1/2})^2 = n - 2R(\mathbb{G}) \\ &= (t(2^t-1)) - \frac{1}{6}\sqrt{6}t(2^(t+1)-1)) \\ &= \frac{1}{6}t(6.2^t - 2\sqrt{6}2^t + \sqrt{6} - 6). \end{aligned}$$

$$\begin{aligned} IRB(\mathbb{G}) &= \sum_{\alpha\beta\in E(\mathbb{G})} (d_\alpha^{1/2} - d_\beta^{1/2})^2 = M_1(\mathbb{G}) - 2RR(\mathbb{G}) \\ &= (10 * t * (2^{(t+1)}-1)) - 2(2sqrt6t(2^{(t+1)}-1)) \\ &= (10t - 4\sqrt{6}t)(2^{t+1}-1). \end{aligned}$$

$$\begin{aligned} IRC(\mathbb{G}) &= \frac{\sum_{\alpha\beta\in E(\mathbb{G})} \sqrt{d_\alpha d_\beta}}{m} - \frac{2m}{n} = \frac{RR(\mathbb{G})}{m} - \frac{2m}{n} \\ &= \frac{2\sqrt{6}t(2^(t+1)-1)}{t(2^t-1)} - \left(\frac{2(t(2^{t+1}-1))}{t(2^t-1)}\right) \end{aligned}$$

$$= \frac{4.2^t + 2\sqrt{6} - \sqrt{6}2^{t+1} - 2}{2^t - 1}.$$

$$\begin{aligned} IRDIF(\mathbb{G}) &= \sum_{\alpha\beta \in E(\mathbb{G})} \left|\frac{d_\alpha}{d_\beta} - \frac{d_\alpha}{d_\beta}\right| \\ &= \left|\frac{4}{6} - \frac{6}{4}\right| t(2^{t+1} - 1) \\ &= (0.8333)t(2^{t+1} - 1). \end{aligned}$$

$$\begin{aligned} IRL(\mathbb{G}) &= \sum_{\alpha\beta \in E(\mathbb{G})} |ln d_\alpha - ln d_\beta| \\ &= |ln4 - ln6| t(2^{t+1} - 1) \\ &= (0.4054)t(2^{t+1} - 1). \end{aligned}$$

$$\begin{aligned} IRLU(\mathbb{G}) &= \sum_{\alpha\beta \in E(\mathbb{G})} \frac{|d_\alpha - d_\beta|}{min(d_\alpha, d_\beta)} \\ &= \frac{|4-6|}{4}(t(2^{t+1} - 1)) \\ &= (0.5)t(2^{t+1} - 1). \end{aligned}$$

$$\begin{aligned} IRLF(\mathbb{G}) &= \sum_{\alpha\beta \in E(\mathbb{G})} \frac{|d_\alpha - d_\beta|}{\sqrt{d_\alpha . d_\beta}} \\ &= \frac{|4-6|}{\sqrt{24}} t(2^{t+1} - 1) \\ &= (0.4082)t(2^{t+1} - 1). \end{aligned}$$

$$\begin{aligned} IRLA(\mathbb{G}) &= \sum_{\alpha\beta \in E(\mathbb{G})} 2\frac{|d_\alpha - d_\beta|}{(d_\alpha + d_\beta)} \\ &= 2\frac{|4-6|}{10} t(2^{t+1} - 1) \\ &= (0.4)t(2^{t+1} - 1). \end{aligned}$$

$$\begin{aligned} IRD1(\mathbb{G}) &= \sum_{\alpha\beta \in E(\mathbb{G})} ln\{1 + |d_\alpha - d_\beta|\} \\ &= ln\{1 + |4-6|\} t(2^{t+1} - 1) \\ &= (1.0986)t(2^{t+1} - 1). \end{aligned}$$

$$\begin{aligned} IRGA(\mathbb{G}) &= \sum_{\alpha\beta \in E(\mathbb{G})} ln\left(\frac{d_\alpha + d_\beta}{2\sqrt{d_\alpha d_\beta}}\right) \\ &= ln\left(\frac{4+6}{2\sqrt{4 \times 6}}\right) t(2^{t+1} - 1) \\ &= (0.0204)t(2^{t+1} - 1). \end{aligned}$$

□

2.4 Graphical Comparison

In this section, we will give the graphical comparison for irregularity indices of three butterfly networks. Different coloures are fixed for different variants of butterfly graphs. The Red colour is fixed for $BF(t)$, Green is for $HCBF(t)$ and Yellow colour is fixed for $TBF(t)$.

3. Conclusion

A structure can be given a single number by employing the topological index. Knowledge of topological indices plays a key role in the link between quantitative structure activity as well as property. In this paper, we investigate the sixteen irregularity invariants for three butterfly networks. The last section is about the graphical comparison of results of these three kind of butterfly networks.

	t=1	t=2	t=3	t=4	t=5
VAR	-6	1.55	3.06	3.58	3.80
AL	6	28	90	248	6.30
IR1	-24	74.66	411.42	132.66	3576.77
IR2	-1.10	0.23	0.61	0.76	0.83
IRF	28	120	372	1008	2540
IRFW	0.83	0.83	0.83	0.83	0.83
IRA	-0.22	3.14	12.62	9.37	26.40
IRB	0.60	2.28	9.09	25.05	63.64
IRC	-1.10	0.23	0.61	0.76	0.83
IRDIF	2.49	11.66	37.49	103.32	262.48
IRL	1.21	5.67	18.24	50.26	127.70
IRLU	1.5	7.0	22.5	62.0	157.5
IRLF	1.22	5.67	18.24	50.26	127.70
IRLA	1.2	5.6	18.0	49.6	126.0
IRD1	3.29	15.38	49.43	136.22	346.05
IRGA	0.06	0.28	0.91	2.52	6.42

Table 6: Irregularity invariants for Butterfly network $TBF(t)$ for different values of t

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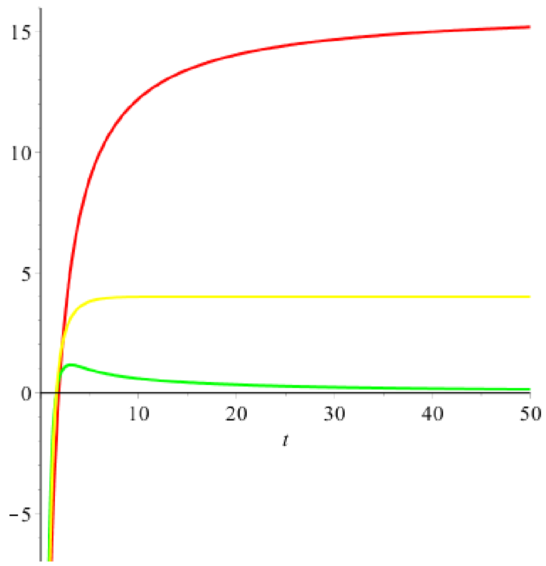
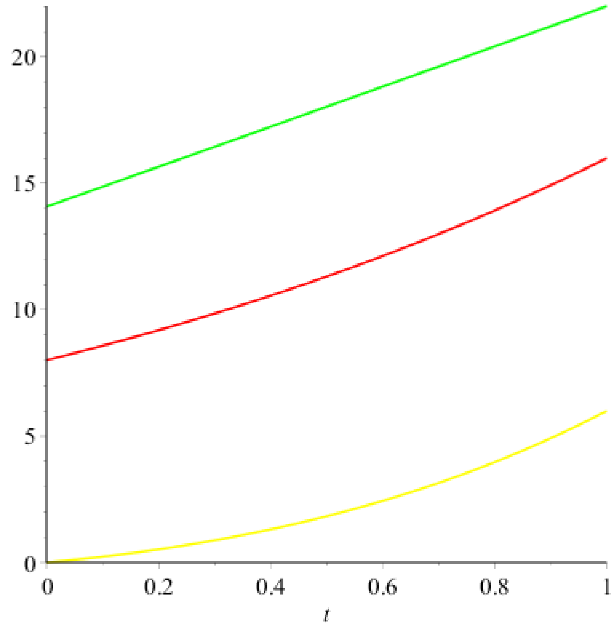
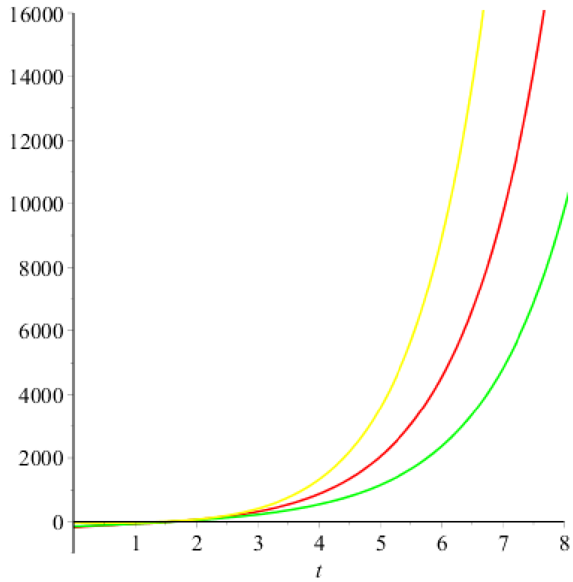


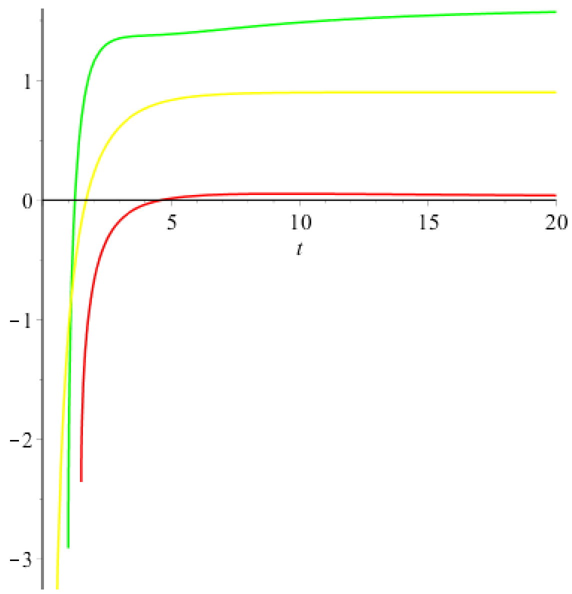
(1) VAR



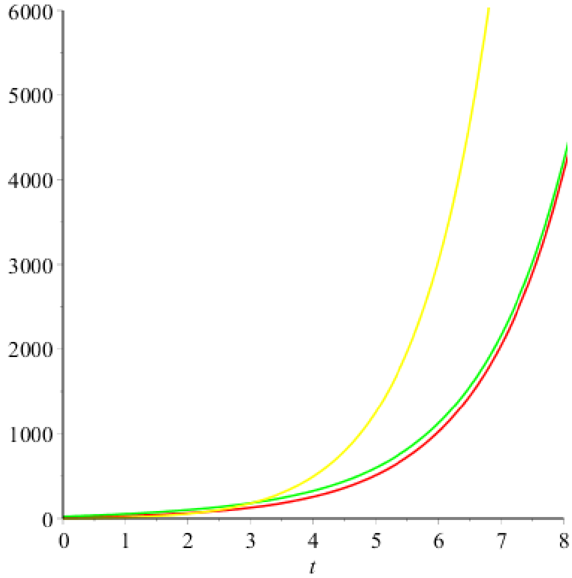
(2) AL



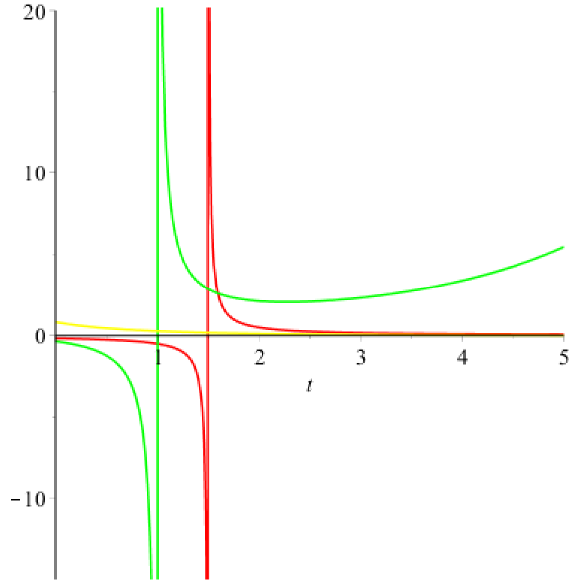
(3) IR1



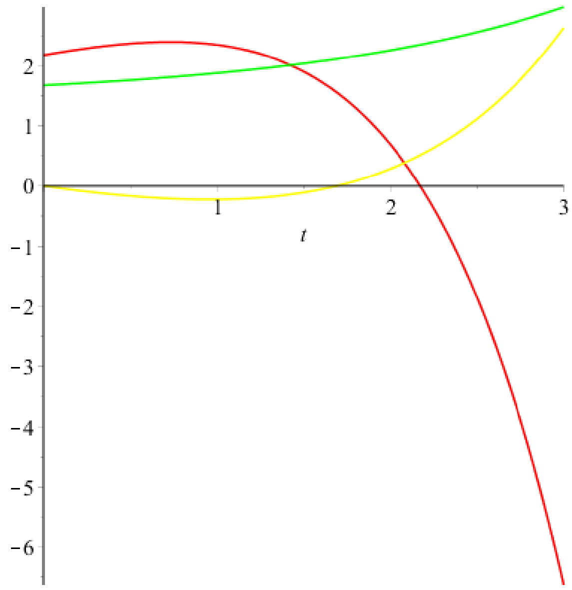
(4) IR2



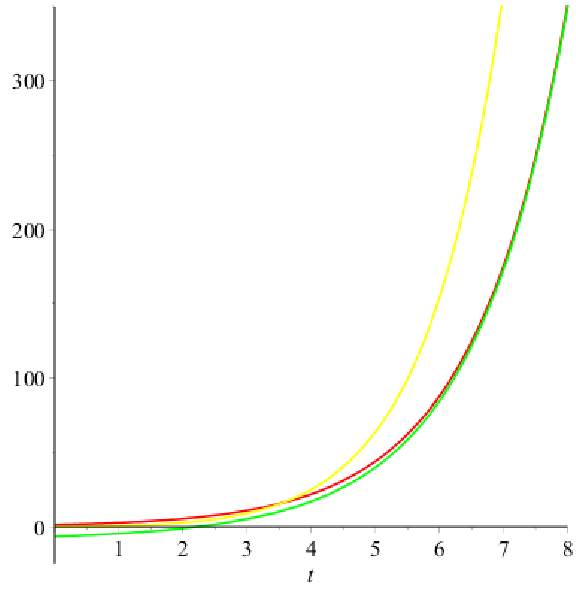
(5) IRF



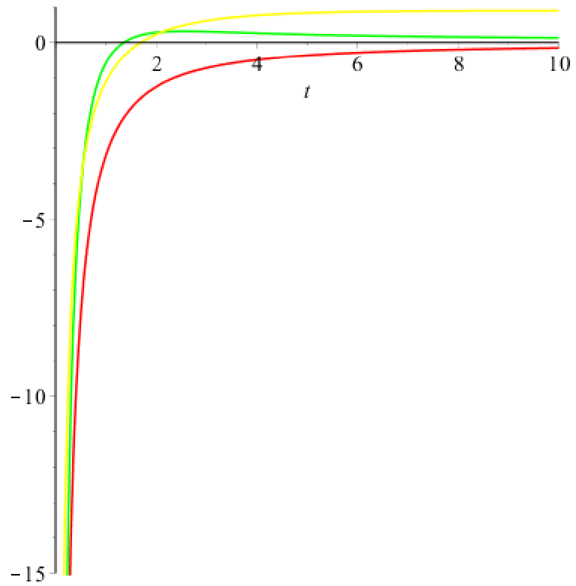
(6) IRFW



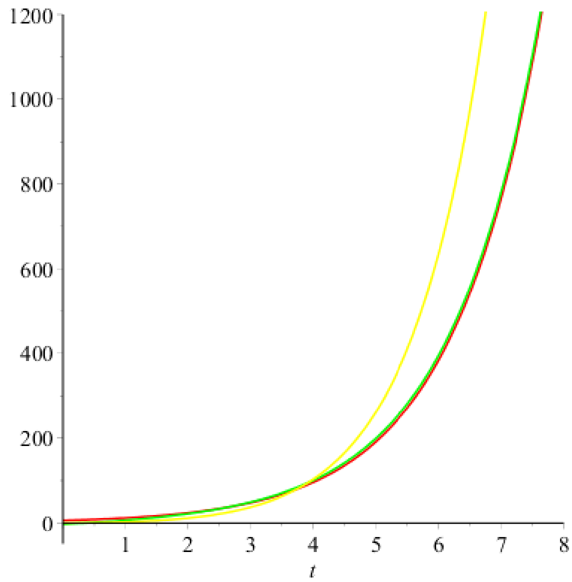
(7) IRA



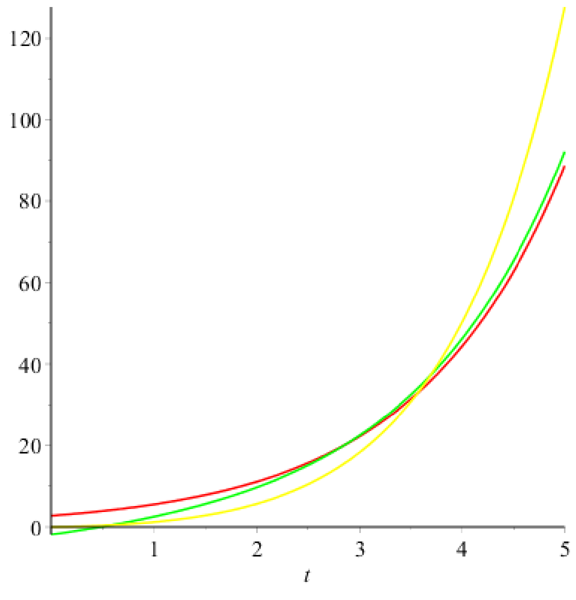
(8) IRB



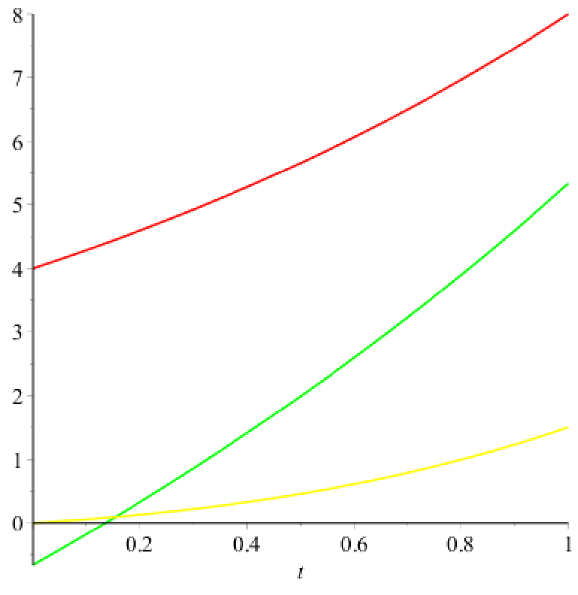
(9) IRC



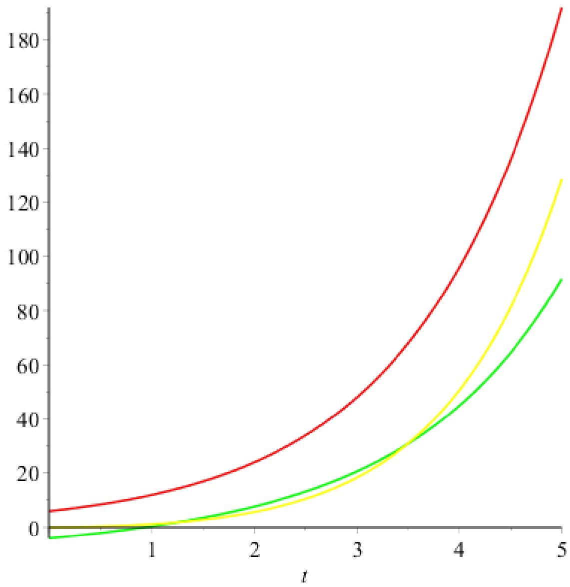
(10) IRDIF



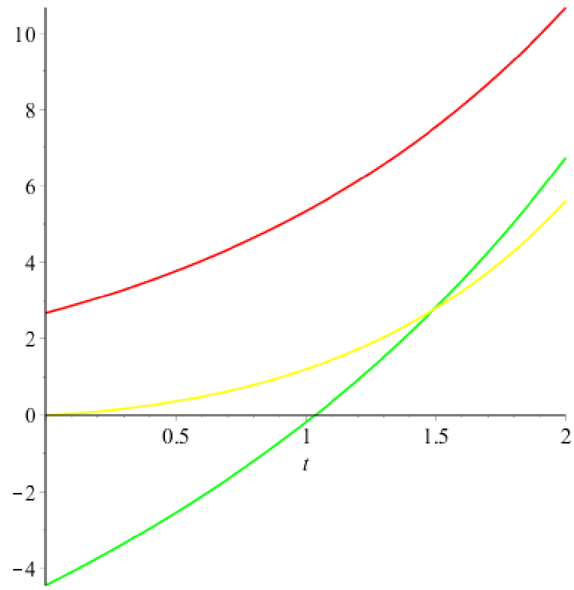
(11) IRL



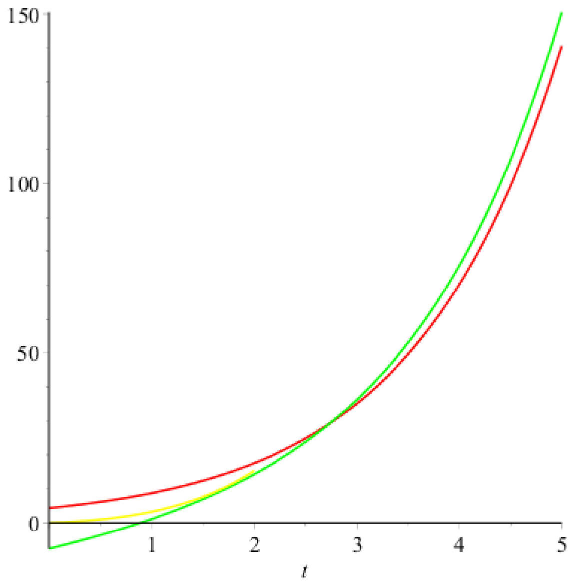
(12) IRLU



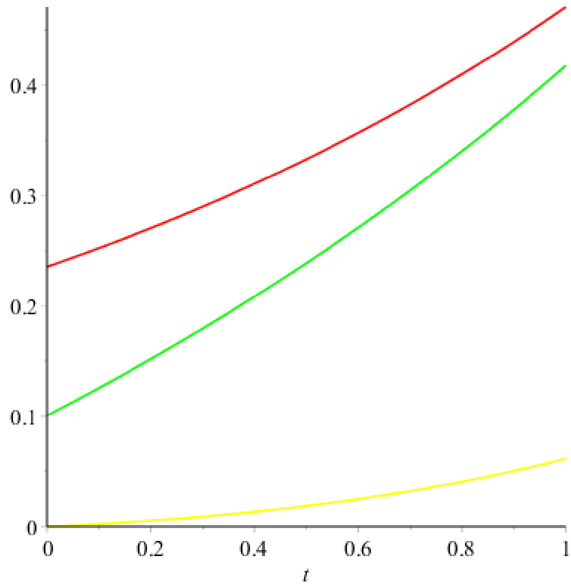
(13) IRLF



(14) IRLA



(15) IRD1



(16) IRGA

Figure 4: Graphical Comparison of Irregularity Butterfly network $BF(t)$, $HCBF(t)$, $TBF(t)$