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Exploring Zig-Zag Triangles: H-Coverings and Practical Applications

Atholl Anderson[1*]

[1] *Australian National University (College of Asia and the Pacific), Australia*

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Abstract

Graph theory is widely used to analyze the structure models in chemistry, biology, computer science, operations research and sociology. Molecular bonds, species movement between regions, development of computer algorithms, shortest spanning tree in a weighted graphs, aircraft scheduling and exploration of diffusion mechanism are some of these structure models. A computing system C can execute an algorithm A if A is graph-isomorphic to a subgraph of C_G. This is equivalent of defining a graph isomorphism between A and subgraph of C_G. This research article gives a small description of graph theory applications and one of graph model (zig-zag triangle) to define a suitable computer algorithm. Let Γ be a connected, simple graph with finite vertices v and edges e. A family $\{\Gamma_1, \Gamma_2, \ldots, \Gamma_p\} \subset \Gamma$ of subgraphs such that for all $e \in E$, $e \in \Gamma_l$, for some l, $l = 1, 2, \ldots, p$ is an *edge-covering* of Γ. If $\Gamma_l \cong \Gamma'$, $\forall l$, then Γ has an Γ'-*covering*. Graph Γ with Γ'-covering is an (a_d, d)-Γ'-antimagic if $f : V \cup E \rightarrow \{1, 2, \ldots, |V| + |E|\}$ a bijection exists and the sum over all vertex-weights and edge-weights of Γ' form a set $\{a_d, a_d + d, \ldots, a_d + (p-1)d\}$. The labeling ξ is *super* for $\xi(V_\Gamma) = \{1, 2, 3, \ldots, |V_\Gamma|\}$ and graph Γ is Γ'-*supermagic* for $d = 0$. This manuscript investigates super (a_d, d)-Γ'-antimagic labelings of zig-zag triangles for differences $d = 1, 2, \ldots, 8$.

Keywords: Computer networks, graphical models, protein-protein networks, biology, chemistry, algorithms, applications, path, cycle, ladder, zig-zag triangle.

1. Introduction and Preliminaries

The research areas of sciences where networks constitute the basic and fundamental study blocks, graph theory (graph labeling, graph coloring etc.) is the most intuitive and fundamental approach to apply and study these sciences. For example: (i) in computer sciences [18], data mining, database designing, image processing, network algorithms, resource allocation, clustering of web documents [16], mobile phone networks(GSM phones) and bi-processor tasks. (ii) in chemistry study of molecular bonds, molecular descriptors, three dimensional complicated simulated structure of atoms and chemoinformatics are some study blocks. (iii) in biology, protein-protein interaction networks, cell biology structure, population genetics, bioinformatics and sequences of cell-samples are some of them. (iv) in operations research, travelling sales man problem, optimization using PERT(Project Evaluation Review Technique), minimum sum coloring, job and time table scheduling [19, 26], game theory. All these applications of graph theory generally requires study and analyzing technique for network algorithms. These algorithms can be one of these:

- Shortest path algorithm
- Planar graphs
- Searching algorithms (DFS, BFS)
- Cycles of different lengths
- Adjacency and Incidence matrices
- Connectedness in a network

Several computer algorithms are being designed to study these types of networks [23, 27]. For example, Python package, GASP, SPANTREE, IGTS, GIRL, GRASPE, AMBIT etc. In this research article, we focus on graphical representation of algorithms [25]. A graph representing an algorithm is called as *facility graph*, where vertices of graphs represents the facilities that execute that algorithm and edges represents the links between these facilities. Let C_G represents the graphical representation of a computing system. A computing system C can execute an algorithm A if A is graph-isomorphic to a subgraph of C_G. This is equivalent of defining a graph isomorphism between A and subgraph of C_G or graph embedding of A in C_G. Several computing systems have been designed using this idea of graph isomorphisms and region adjacency

*Corresponding author (anderson@anu.edu.au)

graphs. For example, (i) the optimal k-FT single loop system [22], fault tolerant computing systems, symbol recognition by error tolerant subgraph [28], automatic channel allocation for wireless networks [17, 23], clustering of web documents [16] and sensor networks [27]. In graph labeling this idea is equivalent of finding an edge-covering of a graph. More specifically, Γ'-covering of a simple graph Γ, where Γ' should be isomorphic to one fixed subgraph Γ" of Γ.

Gutiérrez and Lladó in [11] defined the Γ'-supermagic graphs for $K_{1,m}$, $K_{p,m}$, P_m and for C_m for some subgraph Γ'. Jeyanthi *et al.* [13] proved Γ'-supermagic results for 2-connected graphs, k-polygonal snake and one point union of m-disjoint paths. kC_m-path, $\Gamma \cong K_{1,m} + K_1$, book, ladder related graphs are Γ'-supermagic proved by [7]. Inayah *et al.* [12] introduced Γ'-antimagic graphs with weights forming an arithmetic progression. She also proved some bounds for a_d and d for general graphs and fans. Faisal [33] derived bound for cycle-antimagic labeling of disjoint union of cycles. Recent results on Γ'-antimagic labeling of graphs can be seen in [8, 15, 28–32]. The (super) Γ'-antimagic labeling also related to a (super) d-antimagic labeling of type $(1, 1, 0)$ of a plane graph [3]: a generalization of a face-magic labeling introduced by Lih [5]. Baca *et al.* proved d-antimagic labeling of type $(1, 1, 1)$ for toroidal fullerenes in [4] while in [1] baca *et al.* proved labeling for plane graphs containing Hamiltonian paths.

In this research article, we examined the existence of a super (a_d, d)-Γ'-antimagic labeling for zig-zag triangles.

2. Main Results

A *zigzag-triangle* ZT_m is constructed from path P_m by inserting m new vertices and $3m-2$ new edges. For a zig-zag triangle, $V(ZT_m) = V(P_m) \cup \{b_i : 1 \le i \le m\} = \{a_i, b_i : 1 \le i \le m\}$ and $E(ZT_m) = \{a_ib_i : 1 \le i \le m\} \cup \{a_ia_{i+1}, a_ib_{i+1}, a_{i+1}b_i : 1 \le i \le m-1\}$. Figure 1 represents a zig-zag triangle ZT_m.

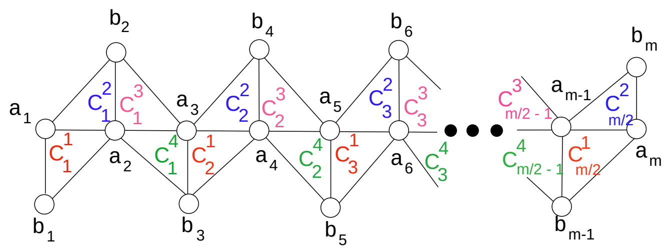

Figure 1: Zig-zag triangle ZT_m.

From figure 1, it is clear that zig-zag triangle has C_3 coverings of type $C_i^{(k)}$, $i = 1, 2, \ldots, \lfloor \frac{m}{2} \rfloor$, $k = 1, 2, 3, 4$, where C_3 is cycle on three vertices. The cycle $C_i^{(1)}$ has the form $\{a_{2i-1}a_{2i}b_{2i-1}a_{2i-1}\}$, $C_i^{(2)}$ has the form $\{a_{2i-1}a_{2i}b_{2i}a_{2i-1}\}$ for $i = 1, 2, \ldots, \lfloor \frac{m}{2} \rfloor$, $C_i^{(3)}$ has the form $\{a_{2i}a_{2i+1}b_{2i}a_{2i}\}$ and $C_i^{(4)}$ has the form $\{a_{2i}a_{2i+1}b_{2i+1}a_{2i}\}$ for $i = 1, 2, \ldots, \lceil \frac{m-1}{2} \rceil$.

Partial sum of the sub-cycle

Under the total labeling f, the C_3-weights of C_i^k's, $i = 1, 2, \ldots, \lfloor \frac{m}{2} \rfloor, k = 1, 2$ are:

$$\begin{aligned} wt_f(C_i^{(1)}) &= f(a_{2i-1}) + f(a_{2i}) + f(b_{2i-1}) + f(a_{2i-1}a_{2i}) + \\ &\quad + f(a_{2i}b_{2i-1}) + f(a_{2i-1}b_{2i-1}) \qquad (1) \\ wt_f(C_i^{(2)}) &= f(a_{2i-1}) + f(a_{2i}) + f(b_{2i}) + f(a_{2i-1}a_{2i}) + \\ &\quad + f(a_{2i}b_{2i}) + f(a_{2i-1}b_{2i}) \qquad (2) \end{aligned}$$

The C_3-weights of $C_i^{(k)}$s for $i = 1, 2, \ldots, \lceil \frac{m}{2} \rceil, k = 3, 4$ are:

$$\begin{aligned} wt_f(C_i^{(3)}) &= f(a_{2i}) + f(a_{2i+1}) + f(b_{2i}) + \\ &\quad + f(a_{2i}a_{2i+1}) + f(a_{2i+1}b_{2i}) + f(a_{2i}b_{2i}) \qquad (3) \\ wt_f(C_i^{(4)}) &= f(a_{2i}) + f(a_{2i+1}) + f(b_{2i+1}) + f(a_{2i}a_{2i+1}) + \\ &\quad + f(a_{2i+1}b_{2i+1}) + f(a_{2i}b_{2i+1}) \qquad (4) \end{aligned}$$

The next theorems prove the existence of a super (a_1, d)-C_3-antimagic labeling for zig-zag triangles where $d \in \{1, 2, \ldots, 8\}$.

Theorem 2.1. *The zigzag-triangle $ZT_m, m \ge 2$ possesses a super $(a_1, 1)$-C_3-antimagic labeling.*

Proof. The total labeling f_d for difference $d = 1$ is defined as:

$$f_1(a_i) = \left\lceil \frac{m}{2} \right\rceil + i, \qquad i = 1, 2, \ldots, m$$

$$f_1(b_i) = \begin{cases} \frac{i+1}{2} & i \equiv 1 \pmod 2 \\ \lceil \frac{3m}{2} \rceil + \frac{i}{2} & i \equiv 0 \pmod 2 \end{cases}$$

$$f_1(a_i b_i) = \begin{cases} 3m - \frac{i-1}{2} & i \equiv 1 \pmod 2 \\ \lceil \frac{5m+1}{2} \rceil - \frac{i}{2} & i \equiv 0 \pmod 2 \end{cases}$$

$$f_1(a_i a_{i+1}) = 3m + i, \quad i = 1, 2, \dots, m-1$$

For $1 \le i \le m-1$

$$f_1(a_i b_{i+1}) = \begin{cases} 5m - i - 1 & i \equiv 1 \pmod 2 \\ 6m - (i+2) & i \equiv 0 \pmod 2 \end{cases}$$

For $2 \le i \le m$

$$f_1(a_i b_{i-1}) = \begin{cases} 5m - i & i \equiv 1 \pmod 2 \\ 6m - i - 1 & i \equiv 0 \pmod 2 \end{cases}$$

Using equation (1–4),

$$\begin{aligned} wt_{f_1}(C_i^{(1)}) &= (2\lceil \frac{m}{2} \rceil + 4i - 1) + (12m - 2) \\ &= 2\lceil \frac{m}{2} \rceil + 12m - 2 + 4i \end{aligned}$$

$$\begin{aligned} wt_{f_1}(C_i^{(2)}) &= (2\lceil \frac{m}{2} \rceil + 4i - 1) + \lceil \frac{8m+1}{2} \rceil + (8m - 1) \\ &= 2\lceil \frac{m}{2} \rceil + \lceil \frac{8m+1}{2} \rceil + 8m + 4i - 2 \end{aligned}$$

$$\begin{aligned} wt_{f_1}(C_i^{(3)}) &= (2\lceil \frac{m}{2} \rceil + 4i) + \lceil \frac{8m+1}{2} \rceil + 8m \\ &= 2\lceil \frac{m}{2} \rceil + \lceil \frac{8m+1}{2} \rceil + 8m + 4i \end{aligned}$$

$$\begin{aligned} wt_{f_1}(C_i^{(4)}) &= (2\lceil \frac{m}{2} \rceil + 4i + 1) + (12m - 1) \\ &= 2\lceil \frac{m}{2} \rceil + 12m + 4i \end{aligned}$$

It is better to consider the C_3-weights in the following order:

- $C_i^{(1)}, \quad i = 1, 2, \dots, \lfloor \frac{m}{2} \rfloor$
- $C_i^{(2)}, \quad i = 1, 2, \dots, \lfloor \frac{m}{2} \rfloor$
- $C_i^{(4)}, \quad i = 1, 2, \dots, \lceil \frac{m-1}{2} \rceil$
- $C_i^{(3)}, \quad i = 1, 2, \dots, \lceil \frac{m-1}{2} \rceil$

We can easily verify that the C_3-weights for $m \equiv 0 \pmod 2$ form the set:

$$\{wt_{f_1}(C_3^{(j)})\} = \{13m + 1 + j\} \quad j = 1, 2, \dots, 2m-1 \tag{5}$$

And for $m \equiv 1 \pmod 2$ form the set:

$$\{wt_{f_1}(C_3^{(j)})\} = \{13m + 2 + j\} \quad j = 1, 2, \dots, 2m-1 \tag{6}$$

Thus the given labeling is a super $(a, 1)$-C_3-antimagic labeling with an initial term $a_1 = 13m + 2$ and a common difference $d = 1$. □

Theorem 2.2. *The zigzag-triangle $ZT_m^2, m \ge 2$ possesses a super (a_d, d)-C_3-antimagic labeling for $d \in \{2, 3, 4, 7\}$.*

Proof. The total labeling f_d is defined as:

$$f_d(a_i) = \begin{cases} \frac{i+1}{2} & i \equiv 1 \pmod 2 \\ \lceil \frac{3m}{2} \rceil + \frac{i}{2} & i \equiv 0 \pmod 2 \end{cases}$$

$$f_d(b_i) = \left\lfloor \frac{n-1}{2} \right\rfloor + i + 1, \quad i = 1, 2, \ldots, m$$

$$f_d(a_i a_{i+1}) = \begin{cases} 2(m+i) & d = 2, 4 \\ 3m + i & d = 3 \\ 2(m+2i) - 1 & d = 7 \end{cases}$$

$$f_d(a_i b_i) = \begin{cases} 2(m+i) - 1 & d = 2, 4 \\ 2m + i & d = 3 \\ 2(m+2i) - 3 & d = 7 \end{cases}$$

For $1 \leq i \leq m-1$

$$f_d(a_i b_{i+1}) = \begin{cases} 2(3m - i - 1) & d = 2 \\ 2(2m + i) - 1 & d = 3, 4 \\ 2(m+2i) & d = 7 \end{cases}$$

For $2 \leq i \leq m$

$$f_d(a_i b_{i-1}) = \begin{cases} 2(3m - i) + 1 & d = 2 \\ 4(m-1) + 2i & d = 3 \\ 2(2m+i) - 4 & d = 4 \\ 2(m - 3 + 2i) & d = 7 \end{cases}$$

Using equation (1–4),

$$wt_{f_d}(C_i^{(k)}) = \begin{cases} \lceil \frac{3m}{2} \rceil + \lfloor \frac{m-1}{2} \rfloor + 2(5m + 4i + k - 3) \\ d = 2, \ k = 1, 2, 3, 4 \\ \lceil \frac{3m}{2} \rceil + \lfloor \frac{m-1}{2} \rfloor + 3(3m + 4i + k - 3) \\ d = 3, \ k = 1, 2, 3, 4 \\ \lceil \frac{3m}{2} \rceil + \lfloor \frac{m-1}{2} \rfloor + 4(2m + k) + 16i - 13 \\ d = 4, \ k = 1, 2, 3, 4 \\ \lceil \frac{3m}{2} \rceil + \lfloor \frac{m-1}{2} \rfloor + 2(3m + 14i) + 7k - 25 \\ d = 7, \ k = 1, 2, 3, 4 \end{cases}$$

It is better to consider the C_3-weights in the following order:

- $C_i^{(1)}, \quad i = 1, 2, \ldots, \lfloor \frac{m}{2} \rfloor$
- $C_i^{(2)}, \quad i = 1, 2, \ldots, \lfloor \frac{m}{2} \rfloor$
- $C_i^{(3)}, \quad i = 1, 2, \ldots, \lceil \frac{m-1}{2} \rceil$
- $C_i^{(4)}, \quad i = 1, 2, \ldots, \lceil \frac{m-1}{2} \rceil$

We can easily verify that the $C_3^{(j)}$-weights for $m \equiv 0 \pmod 2$ form the set:

$$wt_{f_d}(C_3^{(j)}) = \begin{cases} \{12m + 1 + 2j : 1 \leq j \leq 2(m-1)\} \\ d = 2 \\ \{11m + 2 + 3j : 1 \leq j \leq 2(m-1)\} \\ d = 3 \\ \{2(5m + 1 + 2j) : 1 \leq j \leq 2(m-1)\} \\ d = 4 \\ \{2(4m + 1) + 7j : 1 \leq j \leq 2(m-1)\} \\ d = 7 \end{cases} \tag{7}$$

The $C_3^{(j)}$-weights for $m \equiv 1 \pmod 2$ form the set:

$$wt_{f_d}(C_3^{(j)}) = \begin{cases} \{2(6m+1+j) : 1 \le j \le 2(m-1)\} \\ d=2 \\ \{11m+3(1+j) : 1 \le j \le 2(m-1)\} \\ d=3 \\ \{2(5m+2j)+3 : 1 \le j \le 2(m-1)\} \\ d=4 \\ \{8m+3+7j : 1 \le j \le 2(m-1)\} \\ d=7 \end{cases} \tag{8}$$

We can easily verify the C_3-weights from equation (8) as:

$$\begin{aligned} a_2 &= 12m+2, \quad \text{and} \quad d=2 \\ a_3 &= 11m+3, \quad \text{and} \quad d=3 \\ a_4 &= 10m+3, \quad \text{and} \quad d=4 \\ a_7 &= 8m+3, \quad \text{and} \quad d=7 \end{aligned}$$

and thus the given labelings are super (a_d, d)-C_3-antimagic labeling for differences $d \in \{2,3,4,7\}$. □

Theorem 2.3. *The zigzag-triangle $ZT_m^2, m \ge 2$ possesses a super (a_d, d)-C_3-antimagic labeling for $d \in \{5,6,8\}$.*

Proof. For $d \in \{5,6,8\}$ the total labeling f_d is defined as:

$$f_d(a_i) = \begin{cases} i & i \equiv 0 \pmod 2 \ \forall m \\ m-1+i & i,m \equiv 0 \quad (mod\ 2) \\ m+i & i \equiv 0,\ m \equiv 1 \quad (mod\ 2) \end{cases}$$

$$f_d(b_i) = 2i, \qquad i = 1,2,\dots,m$$

$$f_d(a_ia_{i+1}) = \begin{cases} 2(m+i) & d=5 \\ 2(2m+i)-1 & d=6 \\ 2(m+2i)-1 & d=8 \end{cases}$$

$$f_1(a_ib_i) = \begin{cases} 2(m+i)-1 & d=5,6 \\ 2m-3+4i & d=8 \end{cases}$$

For $1 \le i \le m-1$

$$f_d(a_ib_{i+1}) = \begin{cases} 2(2m+i)-1 & d=5 \\ 2(m+2i) & d=6,8 \end{cases}$$

For $2 \le i \le m$

$$f_d(a_ib_{i-1}) = \begin{cases} 4(m-1)+2i & d=5 \\ 2(m-3+2i) & d=6,8 \end{cases}$$

Using equation (1–4), for $m \equiv 0 \pmod 2$

$$wt_{f_d}(C_i^{(k)}) = \begin{cases} 3(3m-6)+5(4i+k) \\ d=5,\ k=1,2,3,4 \\ 3(3m+8i+2k)-22 \\ d=6,\ k=1,2,3,4 \\ 7m-30+8(4i+k) \\ d=8,\ k=1,2,3,4 \end{cases}$$

Using equation (1–4), for $m \equiv 1 \pmod 2$

$$wt_{f_d}(C_i^{(k)}) = \begin{cases} 9m-17+5(4i+k) \\ d=5,\ k=1,2,3,4 \\ 3[3m-7+2(4i+k)] \\ d=6,\ k=1,2,3,4 \\ 7m-29+8(4i+k) \\ d=8,\ k=1,2,3,4 \end{cases}$$

It is better to consider the C_3-weights in the following order:

- $C_i^{(1)}, \quad i = 1, 2, \ldots, \lfloor \frac{m}{2} \rfloor$

- $C_i^{(2)}, \quad i = 1, 2, \ldots, \lfloor \frac{m}{2} \rfloor$

- $C_i^{(3)}, \quad i = 1, 2, \ldots, \lceil \frac{m-1}{2} \rceil$

- $C_i^{(4)}, \quad i = 1, 2, \ldots, \lceil \frac{m-1}{2} \rceil$

We can easily verify that the $C_3^{(j)}$-weights for $m \equiv 0 \pmod 2$ form the set:

$$wt_{f_d}(C_3^{(j)}) = \begin{cases} \{9m + 2 + 5j : 1 \leq j \leq 2(m-1)\} \\ d = 5 \\ \{9m + 2 + 6j : 1 \leq j \leq 2(m-1)\} \\ d = 6 \\ \{7m + 2 + 8j : 1 \leq j \leq 2(m-1)\} \\ d = 8 \end{cases} \tag{9}$$

The $C_3^{(j)}$-weights for $m \equiv 1 \pmod 2$ form the set:

$$wt_{f_d}(C_3^{(j)}) = \begin{cases} \{9m + 3 + 5j : 1 \leq j \leq 2(m-1)\} \\ d = 5 \\ \{9m + 3 + 6j : 1 \leq j \leq 2(m-1)\} \\ d = 6 \\ \{7m + 3 + 8j : 1 \leq j \leq 2(m-1)\} \\ d = 8 \end{cases} \tag{10}$$

From equations (9) and (10) we can easily verify that the C_3-weights as:

$$\begin{aligned} a_2 &= 9m + 3, \quad \text{and} \quad d = 5 \\ a_3 &= 9m + 3, \quad \text{and} \quad d = 6 \\ a_4 &= 7m + 3, \quad \text{and} \quad d = 8 \end{aligned}$$

and thus the given labelings are super (a_d, d)-C_3-antimagic labeling for differences $d \in \{5, 6, 8\}$. $\square$

3. Conclusion

In this manuscript, we provide information related to graphical representation and its usage to other fields: computer science, chemistry, biology, operations research. We write several names and methods of graphical models and their computer algorithms with sufficient references. The bridge between an efficient computer algorithm and graphical representation of network is also defined and discussed in this article. We prove results for a Γ'-antimagic covering of a family *zig-zag triangles* for differences $d \in \{1, 2, \ldots, 8\}$. One can derive results for further differences as well as an efficient computer algorithm based on the Γ'-covering of zig-zag triangle to support ones research in theoretical computer sciences.

4. Conflict of Interests

The author(s) declare that there is no conflict of interests.

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